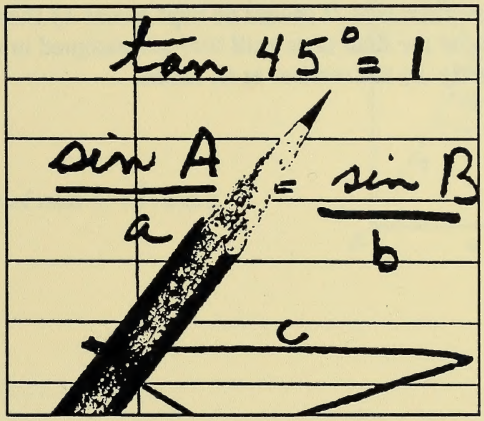
MATHEMATICS 33

Trigonometry



Unit 5

Learning Facilitator's Manual

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 33
Learning Facilitator's Manual
Unit 5
Trigonometry
Alberta Distance Learning Centre
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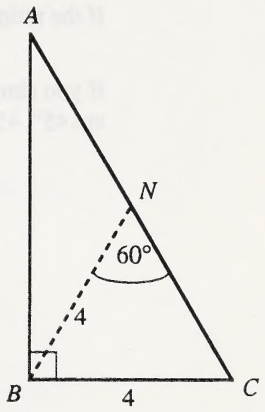
Topic 1: 45° - 45° - 90° and 30° - 60° - 90° Triangles

(8) 1. ABC is a right triangle, where $BN = BC = 4$. If $\angle BNC = 60°$, find the measures of the following. (Note that $\angle$ = angle)



a. AB

Since $BN = BC$, $\angle BNC = \angle NCB$. Therefore, $\angle NCB = 60°$.

$$\begin{aligned}\angle BAC &= 180° - (90° + 60°)\\ &= 180° - 150°\\ &= 30°\end{aligned}$$

A 30° - 60° - 90° triangle has sides of ratio 1, $\sqrt{3}$, and 2.
Since $BC = 4$, side $AB = 4\sqrt{3}$.

b. AC

Since $BC = 4$, side $AC = 8$.

c. AN

$$\begin{aligned}\angle CBN &= 180° - (60° + 60°)\\ &= 180° - 120°\\ &= 60°\end{aligned}$$

Since $\angle CBN = \angle NCB$, $BN = NC$. Therefore, $NC = 4$.

$$\begin{aligned}AN &= AC - NC\\ &= 8 - 4\\ &= 4\end{aligned}$$

d. $\angle ABN$

$$\begin{aligned}\angle ABN &= 90° - \angle CBN\\ &= 90° - 60°\\ &= 30°\end{aligned}$$

(2)

2. If the measures of the sides of a triangle are $\frac{3}{2}$, $\frac{3}{2}$, and $\frac{3\sqrt{2}}{2}$ respectively, what are the measures of its angles?

If the ratios of the sides of a triangle are $\frac{3}{2}$, $\frac{3}{2}$, and $\frac{3\sqrt{2}}{2}$, the triangle is isosceles.

If you simplify the ratio, you get 1, 1, and $\sqrt{2}$. This is the ratio for a right triangle whose angles are 45°, 45°, and 90°.

Topic 1

_____ marks

Topic 2: Trigonometric Ratios in the Coordinate Plane

(2) 1. Angles with the same initial sides and the same terminal sides are said to be

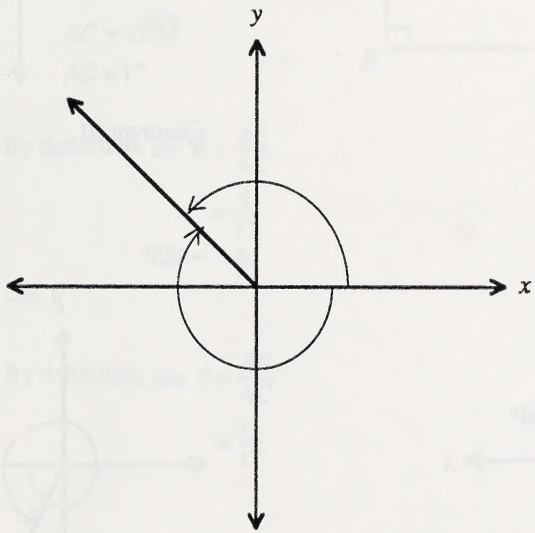
A. collinear

(B.) coterminal

C. concurrent

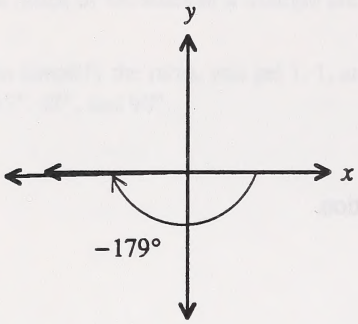
D. equal

The correct answer is B. This is by definition.



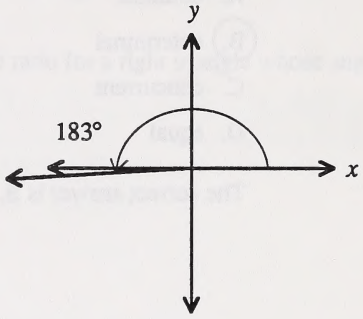
(8) 2. Draw a diagram to show in which quadrant the terminal arm will lie for the following angles.

a. $-179°$



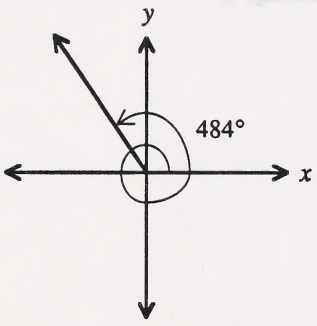
Quadrant III

b. $183°$



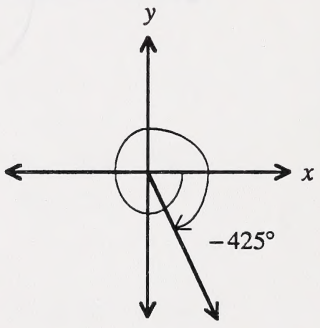
Quadrant III

c. $484°$



Quadrant II

d. $-425°$



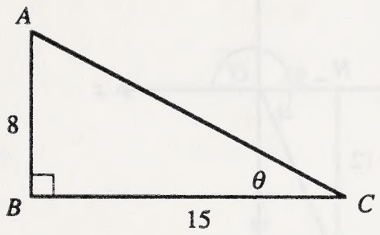
Quadrant IV

(5) 3. If $\tan\theta = \frac{8}{15}$, what is the value of the following? Show all your work.

a. $\sin\theta$

If $\tan\theta = \frac{8}{15}$, then by the Pythagorean theorem, $AC^2 = AB^2 + BC^2$.

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 8^2 + 15^2$$
$$AC^2 = 64 + 225$$
$$AC^2 = 289$$
$$AC = \sqrt{289}$$
$$AC = 17$$



By definition, $\sin\theta = \frac{AB}{AC}$

$$= \frac{8}{17}$$

b. $\cos\theta$

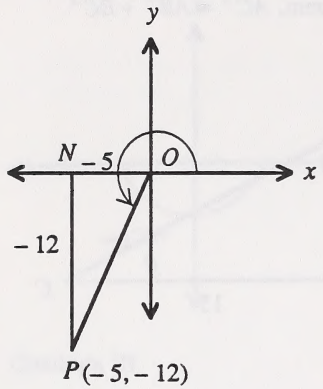
By definition, $\cos\theta = \frac{BC}{AC}$

$$= \frac{15}{17}$$

(5)

4. Find the primary ratios for an angle θ when its terminal arm passes through the point $P(-5,-12)$. Then find θ.

Since the terminal arm passes through $P(-5,-12)$, use the Pythagorean theorem.



$$\begin{aligned} OP^2 &= ON^2 + NP^2 \\ &= (-5)^2 + (-12)^2 \\ &= 25 + 144 \\ &= 169 \\ \therefore OP &= 13 \end{aligned}$$

The primary ratios are sine, cosine, and tangent.

By definition, $\sin\theta = \dfrac{-12}{13}$,

$\cos\theta = \dfrac{-5}{13}$,

and $\tan\theta = \dfrac{-12}{-5} = \dfrac{12}{5}$

If $\tan\theta = \frac{12}{5} = 2.4$, then $\theta = \tan^{-1} 2.4$

$\doteq 67.38° + 180°$

$\doteq 247.38°$.

④ 5. Use a scientific calculator to find the following. Round your answers to four decimal places.

a. cos 42.4°

Enter	Display
42.4	42.4
cos	0.7385

cos 42.4° = 0.7385

b. tan 75.75°

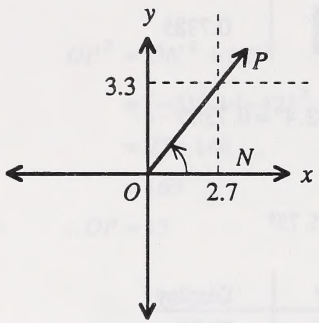
Enter	Display
75.75	75.75
tan	3.9375

tan 75.75° = 3.9375

⑥ 6. Point $P(2.7, 3.3)$ is in the first quadrant. If PO forms an angle θ with the x-axis, calculate the following to three decimal places.

a. cot θ

Use the Pythagorean theorem to find OP.



$$OP^2 = ON^2 + NP^2$$
$$OP^2 = 2.7^2 + 3.3^2$$
$$OP^2 = 7.29 + 10.89$$
$$OP^2 = 18.18$$
$$\therefore OP \doteq 4.2638$$

By definition, $\cot\theta = \frac{ON}{NP}$

$$= \frac{2.7}{3.3}$$
$$\doteq 0.818$$

b. sec θ

By definition, $\sec\theta = \frac{OP}{ON}$

$$= \frac{4.2638}{2.7}$$
$$\doteq 1.579$$

c. csc θ

By definition, $\csc\theta = \frac{OP}{NP}$

$$= \frac{4.2638}{3.3}$$
$$\doteq 1.292$$

(2) 7. If $\cos \theta = 0.9905, 0 < \theta < 90°$. What is the value of θ?

A. $-7.6°$

B. $7.7°$

C. $-7.8°$

(D.) $7.9°$

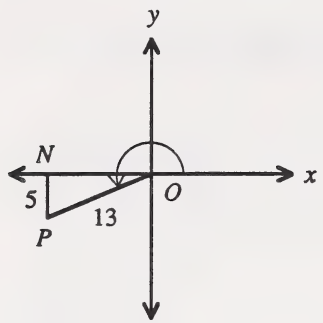
The correct response is D.
For verification, use a calculator.

Enter	Display
7.9	7.9
cos	0.990509463

$\theta = 7.9°$

(5) 8. If $\sin\theta = -\frac{5}{13}$, and $\cos\theta$ is negative, do the following.

a. Find $\tan\theta$ to four decimal places.



Since both sine and cosine are negative, the angle θ lies in the third quadrant. Use the Pythagorean theorem to find ON.

$$ON^2 = OP^2 - PN^2$$
$$ON^2 = 13^2 - 5^2$$
$$ON^2 = 169 - 25$$
$$ON^2 = 144$$
$$\therefore ON = 12$$

By definition, $\tan\theta = \frac{NP}{NO}$

$$= \frac{5}{12}$$
$$\doteq 0.4167$$

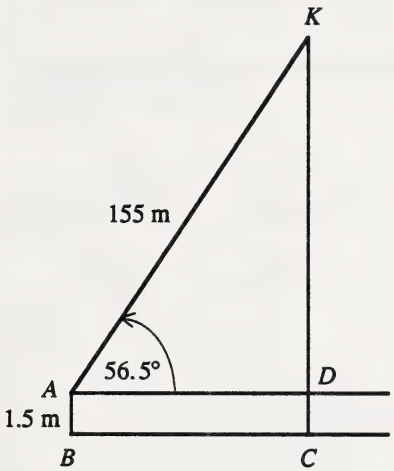
b. What is the value of θ? Round your answer to one decimal place.

Tangent is positive in the third quadrant. Since $\tan\theta \doteq 0.4167$, the angle $\doteq 22.6°$ (from the table or calculator). The angle θ lies in the third quadrant.

Thus, $\theta \doteq 180° + 22.6°$
$\doteq 202.6°$

⑥ 9. A girl is flying a kite with a string that is 155 m long. Assume that the string is straight and the angle of elevation to the kite is 56.5°. If the girl is holding the string from a height of 1.5 m above the ground, calculate the height of the kite.

Length of string = 155 m
Angle of elevation = 56.5°
Height of string above the ground = 1.5 m



$$\sin 56.5° = \frac{KD}{AK}$$

$$\sin 56.5° = \frac{KD}{155}$$

$$KD = 155 \sin 56.5°$$

$$KD \doteq 129.2523$$

$$DC = AB = 1.5 \text{ m}$$

Let the height of the kite be KC.

$$KC = KD + DC$$
$$\doteq 129.2523 + 1.5$$
$$\doteq 130.7523$$

The kite is approximately 130.75 m above the ground.

④ 10. Give the exact value of the following.

a. $-\sin -(180°-45°)$

$$\begin{aligned}-\sin -(180°-45°) &= \sin(180°-45°)\\ &= \sin(135°)\\ &= \sin\ 45°\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

b. $\cos\ 450°$

$$\begin{aligned}\cos\ 450° &= \cos(360°+90°)\\ &= \cos\ 90°\\ &= 0\end{aligned}$$

c. $\tan -225°$

$$\begin{aligned}\tan(-225°) &= \tan\ 135°\\ &= -\tan\ 45°\\ &= -1\end{aligned}$$

d. $-\cot(360-240°)$

$$\begin{aligned}-\cot(360°-240°) &= -\cot(120°)\\ &= \cot\ 60°\\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

(3) 11. Which of the following statements is true for $0 < \theta < 90$?

A. $\sin(180° - \theta) = \sin(180° + \theta)$

(B.) $\cos(180° - \theta) = \cos(180° + \theta)$

C. $\sin(180° + \theta) = \cos(180° + \theta)$

D. $\cos(180° - \theta) = \sin(180° - \theta)$

The correct choice is B. Although the angles end up in different quadrants, the value is the same in sign and magnitude.

LS	RS
$\cos(180° - \theta)$	$\cos(180° + \theta)$
$-\cos\ \theta$	$-\cos\ \theta$

LS = RS

Topic 2

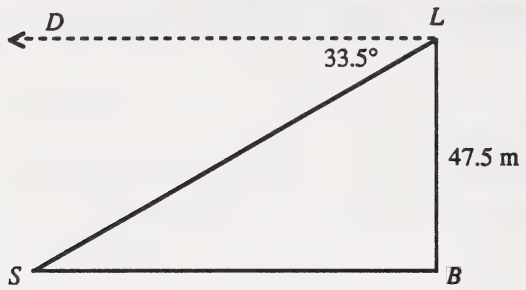
_____ marks

Topic 3: Sine and Cosine Laws

(5)

1. The angle of depression of a ship from the top of a 47.5 m lighthouse is 33.5°. What is the distance from the ship to the foot of the lighthouse? Round your answer to two decimal places.

Height of lighthouse = 47.5 m
Angle of depression of lighthouse = 33.5°
Let LS be the distance from the top of the lighthouse to the ship.
The angle of elevation is equal to the angle of depression.



With reference to the diagram, $\angle BSL = \angle DLS = 33.5°$.

$$\tan 33.5° = \frac{47.5}{SB}$$

$$SB \doteq \frac{47.5}{0.6619}$$

$$SB \doteq 71.76$$

The distance from the ship to the foot of the lighthouse is approximately 71.76 m.

(5) 2. An 18.4 m oak tree casts a shadow of 25.3 m at 1:00 p.m. What is the angle of elevation of the sun at that time?

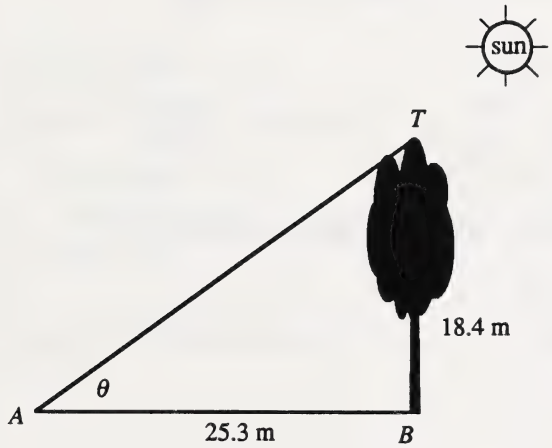
(A.) 36°

B. 20°

C. 18°

D. 72°

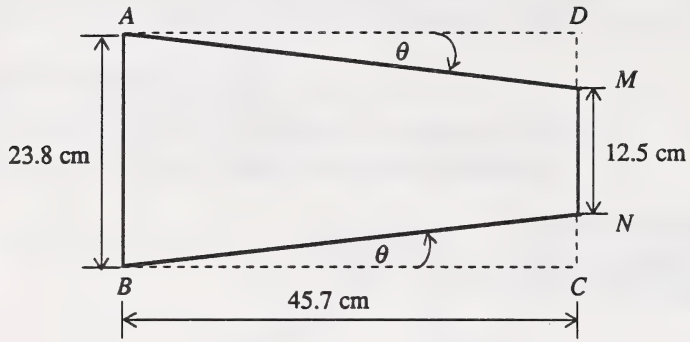
Height of oak tree = 18.4 m
Length of shadow = 25.3 m
Let the angle of elevation equal θ.



$$\tan \theta = \frac{BT}{AB}$$
$$= \frac{18.4}{25.3}$$
$$\doteq 0.7273$$
$$\theta \doteq 36°$$

The correct response is A.

⑥ 3. A machinist is making a shaft according to the dimensions shown. What is the measure of the angle θ if $DM = NC$? Give your answer to the nearest degree.



Length of shaft $AD = 45.7$ cm
Diameter of shaft $AB = 23.8$ cm
Tapered end of shaft $MN = 12.5$ cm
Let θ be the angle of incline of the shaft.

Difference between large and small end of shaft is as follows:

$$AB - MN = 23.8 - 12.5$$
$$= 11.3$$

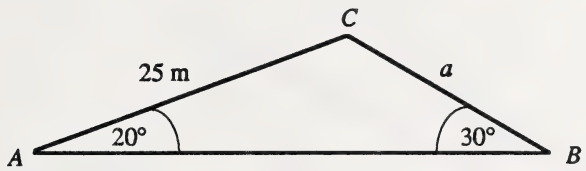
Therefore, $DM = \frac{11.3}{2}$
$$= 5.65$$

Angle of taper $\angle DAM = \theta$

$$\tan \theta = \frac{DM}{DA} = \frac{5.65}{45.7}$$
$$\doteq 0.1236$$
$$\therefore \theta \doteq 7° \quad \text{(from the table or calculator)}$$

The measure of θ is approximately 7°.

(5) 4. Use the sine law to find the length of BC if $\angle ABC = 30°$, $AC = 25$ m, and $\angle BAC = 20°$. Round your answer to two decimal places.



The sine law is as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 20°} = \frac{25}{\sin 30°}$$

$$a = BC = \frac{25 \sin 20°}{\sin 30°} \quad \left(\sin 30° = \frac{1}{2}\right)$$

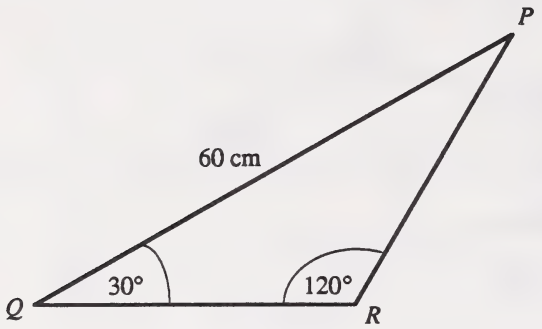
$$BC = 50 \sin 20°$$

$$BC \doteq 50(0.3420)$$

$$BC \doteq 17.10$$

The length of BC is approximately 17.10 m.

④ 5. In ΔPQR, $\angle PQR = 30°$, $\angle PRQ = 120°$, and $PQ = 60$ cm.
Find the measure of the following.



a. PR

Use the sine law.

$$\frac{PR}{\sin 30°} = \frac{60}{\sin 120°}$$

$$PR = \frac{60 \sin 30°}{\sin 120°}$$

$$= \frac{60(0.5)}{\sin 60°} \qquad (\sin 120° = \sin 60°)$$

$$= \frac{30}{\frac{\sqrt{3}}{2}}$$

$$= \frac{60}{\sqrt{3}}$$

$$= \frac{60\sqrt{3}}{3}$$

$$= 20\sqrt{3}$$

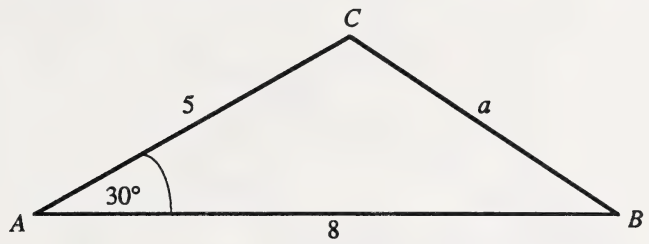
PR is $20\sqrt{3}$ cm or approximately 34.64 cm.

b. QR

Since $\angle Q = 30°$ and $\angle R = 120°$, then $\angle P = (180° - 150°) = 30°$.

Thus, ΔPQR is isosceles and $PR = QR = 20\sqrt{3}$ or approximately 34.64 cm.

(5) 6. ΔABC is an obtuse triangle, where $AB = 8$, $AC = 5$, and $\angle BAC = 30°$.



Find the measure of the following.

a. BC

The cosine law is as follows:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

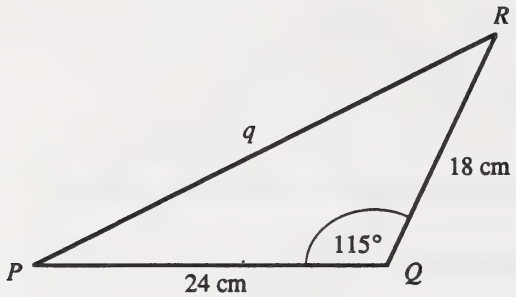
$$a^2 = 5^2 + 8^2 - 2(5)(8)\cos 30°$$
$$a^2 = 25 + 64 - 80\cos 30°$$
$$a^2 = 89 - \frac{80\sqrt{3}}{2}$$
$$a^2 = 89 - 40\sqrt{3}$$
$$a^2 \doteq 89 - 69.28$$
$$a^2 \doteq 19.72$$
$$a \doteq 4.44 \qquad \therefore BC \doteq 4.44$$

b. $\angle ABC$

Use the sine law.

$$\frac{5}{\sin B} \doteq \frac{4.44}{\sin 30°}$$
$$\sin B \doteq \frac{5\sin 30°}{4.44}$$
$$\sin B \doteq \frac{2.5}{4.44}$$
$$\sin B \doteq 0.5631$$
$$\angle B \doteq 34.27°$$

(5) 7. If $PQ = 24$ cm, $QR = 18$ cm, and $\angle PQR = 115°$, use the law of cosines to find the measure of the following.



a. PR

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$q^2 = 18^2 + 24^2 - 2(18)(24) \cos 115°$$
$$q^2 = 324 + 576 - 864(-\cos 65°)$$
$$q^2 \doteq 324 + 576 + 864(0.4226)$$
$$q^2 \doteq 900 + 365.142$$
$$q^2 \doteq 1265.142$$
$$q \doteq 35.569 \qquad \therefore PR \doteq 35.57$$

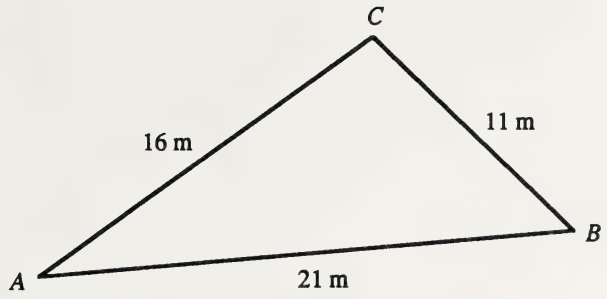
The measure of PR is about 35.57 cm.

b. $\angle RPQ$

$$\frac{18}{\sin P} \doteq \frac{35.57}{\sin 115°}$$
$$\sin P \doteq \frac{18 \sin 115°}{35.57}$$
$$\sin P \doteq \frac{18 \sin 65°}{35.57}$$
$$\sin P \doteq \frac{18(0.9063)}{35.57}$$
$$\sin P \doteq \frac{16.314}{35.57}$$
$$\sin P \doteq 0.4586$$
$$\therefore \angle P \doteq 27.3°$$

The measure of $\angle RPQ$ is approximately 27.3°.

(5) 8. Find $\angle A$ in ΔABC if $AB = 21$ m, $BC = 11$ m, and $CA = 16$ m.



Since the cosine law states that $a^2 = b^2 + c^2 - 2bc \cos A$, you get the following:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{16^2 + 21^2 - 11^2}{2(16)(21)}$$
$$= \frac{256 + 441 - 121}{32(21)}$$
$$= \frac{256 + 320}{672}$$
$$= \frac{576}{672}$$
$$\doteq 0.8571$$
$$\angle A \doteq 31°$$

The angle at A is about 31°.

Topic 3

_____ marks